# DEPARTMENT OF CIVIL ENGINEERING

B.Tech. 4<sup>th</sup> SEMESTER

Fluid Flow in Pipes and Channels

Chapter 1

**Open Channel Flow** 

**Teacher Concerned** 

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## **Open-Channel Flow**

Open channel flows are characterized by a free surface which is exposed to the atmosphere. The pressure on this boundary thus remains approximately constant irrespective of any changes in the water depth and the flow velocity. These free-surface flows occur commonly in engineering practice, and include both large-scale geophysical flows (rivers and estuaries) and small-scale man-made flows (irrigational channels, drainage channels and sewers). Although many of the traditional examples are of primary interest to the civil engineer, the underlying theory of open channel hydraulics is appropriate to any free-surface flow. In general, these flows may be laminar or turbulent, steady or unsteady, and uniform or varied. However, as in the case of pipe flow, this general class of problems may be subdivided into two distinct groups. The first involves significant changes in the water depth over relatively short channel lengths. These are classified as "rapidly varying flows," and are largely unaffected by shear forces. In contrast, the second group involves less rapid changes in the water depth (occurring over longer distances), and are classified as "gradually varying flows." This latter group may be significantly affected by shear forces.

Most elementary open channel hydraulics is based upon three underlying assumptions: (a) the fluid is homogeneous and incompressible; (b) the flow is steady; and (c) the pressure distribution is hydrostatic at all control sections. Although there are important exceptions, notably the inhomogeneity caused by the entrainment of air in a high-speed flow or the unsteadiness associated with the propagation of flood waves or tidal bores, these assumptions are widely applicable and lead to important simplifications of the conservation equations. In particular, if the pressure distribution is assumed hydrostatic at all control sections, this implies that the *streamlines* are straight, parallel, and approximately horizontal, and that there are no pressure gradients due to the curvature of the flow. In this case the hydraulic gradient line, or *piezometric line*, is the same for all streamtubes and is co-incident with the free surface. This result accounts for the wide application of the energy line — hydraulic gradient line as a means of describing an open channel flow.

To emphasize the importance of the unconstrained free-surface boundary, a transitional flow involving a step in a rectangular channel is considered (Figure 1a).

If the volume flow per unit width is  $\frac{1}{V}$ , mass conservation defines the velocity (u) in terms of the water depth (d) such that  $\frac{1}{V} = u_1 d_1 = u_2 d_2$ . Furthermore, if the energy head (E) is expressed in terms of the

specific head (or "specific energy") measured relative to the channel bed, E may be defined in terms of d and  $^{\vec{V}}$  alone:

$$E = d + \frac{\dot{V}^2}{2gd^2},$$

where the final term is an alternative representation of the velocity head. Figure 1b concerns the variation of E and d for a given value of  $\forall$ , and demonstrates that if E and  $\forall$  are fixed there are (in general) two potential solutions for d. In the present case if there are no energy losses, Bernoulli's Theorem gives  $E_2 = E_1 - \Delta z$ , and thus the state ( $E_2$ ,  $d_2$ ) could be represented by point B or point B' on the specific energy curve.



Figure 1a. Transitional flow.



Figure 1b. Specific energy curve (not to scale).

The depths  $d_2$  and d' both represent physically realistic solutions, and are often referred to as "alternate depths" corresponding to two different flow regimes. Although the specific energy in each case is the same (E<sub>2</sub>), point B' corresponds to a deep slow flow whereas point B' describes a shallow fast flow.

Point C on the specific energy curve (Figure 1b) defines the common boundary of these two flow regimes, and represents the so-called critical flow. This state, which is usually defined in terms of a critical depth ( $d_c$ ), represents the minimum specific energy for a given volume discharge. If the water depth increases above the critical depth ( $d > d_c$ ) the specific energy also increases due to the flowwork associated with the hydrostatic pressure. In contrast, if the water depth reduces below the critical depth ( $d < d_c$ ) the kinetic energy (or velocity head) accounts for the increase in the specific energy. The critical flow may also be interpreted as producing the maximum flow for a given specific energy.

If dE/dy = 0 at the critical limit, it follows that the critical depth ( $d_c$ ) and the critical velocity ( $u_c$ ) are given by:

$$d_c = \frac{2}{3}E$$
,  $u_c = \sqrt{gd_c}$ .

These definitions allow the classification of the flow regimes noted above. If  $d > d_c$  (or  $u < u_c$ ) the regime is described as subcritical (or *subundal*) flow; whereas if  $d < d_c$  (or  $u > u_c$ ) supercritical (or

*superundal*) flow is said to occur. A close analogy exists between these definitions of an open channel flow and the distinction of subsonic or supersonic flow in a compressible fluid. Indeed, this analogy can be further extended since the critical velocity  $u_c$  defines the speed of a surface wave in water of depth  $d_c$  As a result the Froude Number (Fr), defined by  $Fr^2 = u/(gd)$ , defines the ratio of the free stream velocity to the surface wave velocity. In the context of open channel flows Fr < 1 implies subcritical flow, Fr > 1 supercritical flow, and Fr = 1 critical flow. Returning to the transition problem (Figure 1a), the development of the water surface over the downstream step is dependent upon the "accessibility" of the two flow regimes. The specific energy curve (Figure 1b) provides guidance in this respect. If the upstream state is defined by A, and the discharge per unit width is constant, any changes must take place along the E-d curve shown on Figure 1b. To move from A to B is clearly possible, but to move from B to B' requires a reduction in the specific energy below  $E_2$ , and thus cannot be justified in the present transition.

As a result, the specific energy curve suggests that if the upstream flow is subcritical (point A), an increase in the bed elevation will produce a reduction in the water depth from d1 to d<sub>2</sub> Not only is this result somewhat unexpected, the accessibility of the various flow regimes is dependent upon the upstream condition. For example, if the initial conditions were described by A' (rather than A) the upstream regime would be supercritical. In this case, a similar accessibility argument suggests that an increase in the bed elevation would produce an increase in the water depth from  $d'_1$  to  $d'_2$ , with the final energy state represented by B' on the E-d curve.

We have already noted that the distinction between subcritical and supercritical flow is dependent upon the velocity of a surface wave or disturbance. This has important implications for the control of any open channel flow, and in particular the estimation of water levels for a given volume discharge. If a flow is supercritical, a disturbance at the water surface is unable to travel upstream (relative to a stationary observer) because the velocity of the flow exceeds the wave velocity (Fr > 1). As a result, all supercritical flows are controlled from upstream, and may be considered "blind" to any changes which arise downstream. In contrast, subcritical flows (Fr < 1) are such that a surface disturbance can either travel upstream or downstream, and as a result these flows are typically controlled from downstream.

In its simplest form, a control structure is designed to change the water depth to (or through) the critical depth ( $d_c$ ), so that the discharge is fixed relative to the depth. In practice, most control structures accelerate a subcritical flow, through the critical regime, to produce a shallow fast supercritical flow. The most common examples of such structures include sluice gates and Weirs (Figure 2a and b).



Figure 3. Open channel control structures.

To include more than one effective control within an open channel, the supercritical flow produced by an upstream control must be reconverted to a subcritical flow. This is usually achieved by a *hydraulic jump* (or stationary bore) in which the characteristics of the subcritical flow are determined by a second downstream control. These events are associated with large energy losses, and are often used as an effective means of dissipating unwanted kinetic energy downstream of an overflow (spillway) or underflow (sluice gate) structure.

#### The Hydraulic Jump or Standing Wave:

We know that for a given discharge per unit width of a channel, for a given value of the specific energy head E there can be two possible depths of flow  $d_1$  and  $d_2$ .



For instance corresponding to specific energy head E = OG [Fig. 14.82], the depth of flow can be  $d_1 = GH$  or  $d_2 = GI$ . The depth  $d_1$  is less than the critical depth and the depth  $d_2$  is greater than the critical depth.

When the depth of flow is  $d_1$  (less than critical depth) the flow is a shooting flow. When the depth of flow is  $d_2$  (greater than critical depth) the flow is a streaming flow. Shooting flow is an unstable type of flow. If due to certain forced situation, a shooting flow exists in a certain region, it will ultimately convert itself into the stable streaming flow on the downstream side. During such a transformation there will occur a sudden rise in water surface. Such a sudden rise in water surface is called a standing wave or a hydraulic jump.

Note. For a hydraulic jump to occur, the existing flow should be a shooting flow i.e., the depth of flow should be less than the critical depth or the Froude number should be greater than 1.



Fig. 14.83.

### Depth after Hydraulic Jump in a Rectangular Channel:

Let q be the discharge per unit width of channel. Consider sections 1-1 and 2-2 before and after hydraulic jump. Let  $d_1$  and  $d_2$  be the depths at these sections. Let  $v_1$  and  $v_2$  be the velocities at these sections. Considering unit width of channel,

Pressure force at section 1-1  $= P_{1} = \frac{wd_{1}^{2}}{2}$ Pressure force at section 2-2  $= P_{2} = \frac{wd_{2}^{2}}{2}$   $= P_{2} - P_{1} = \frac{wd_{2}^{2}}{2} - \frac{wd_{1}^{2}}{2} = \frac{w}{2}(d_{2}^{2} - d_{1}^{2})$ But force  $= [Mass flowing per second] \times [Change in velocity]$   $= \frac{wq}{g}(v_{1} - v_{2})$ 

$$\therefore \qquad \frac{w}{2}(d_2^2 - d_1^2) = \frac{wq}{g}(v_1 - v_2)$$

But

But 
$$v_1 = \frac{q}{d_1}$$
 and  $v_2 = \frac{q}{d_2}$   
 $\therefore \qquad \frac{w}{2}(d_2^2 - d_1^2) = \frac{wq^2}{g} \left(\frac{1}{d_1} - \frac{1}{d_2}\right)$ 

 ${d_2}^2 + d_1 d_2 - \frac{2q^2}{gd_1} = 0$ 

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Solving as a quadratic in  $d_2$ ,

$$d_{2} = \frac{-d_{1} + \sqrt{d_{1}^{2} + \frac{8q^{2}}{gd_{1}}}}{2}$$
$$d_{2} = -\frac{d_{1}}{2} + \sqrt{\frac{d_{1}^{2} + \frac{2q^{2}}{gd_{1}}}{4 + \frac{2q^{2}}{gd_{1}}}} \qquad \dots (ii)$$

Since

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$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2v_1^2 d_1}{g}} \qquad \dots (iii)$$

From equation (ii) or (iii) the depth  $d_2$  after the hydraulic jump can be calculated.

 $q = v_1 d_1$ 

Note. If the depth  $d_2$  after the hydraulic jump is known, the depth  $d_1$  before the hydraulic jump can be determined by the rotation

$$d_{1} = -\frac{d_{2}}{2} + \sqrt{\frac{d_{2}^{2}}{4} + \frac{2q^{2}}{gd_{2}}}$$
$$d_{1} = -\frac{d_{2}}{2} + \sqrt{\frac{d_{2}^{2}}{4} + \frac{2v_{2}^{2}d_{2}}{g}}$$

or,

Loss of energy due to hydraulic jump

$$h_{i} = \left(d_{1} + \frac{v_{1}^{2}}{2g}\right) - \left(d_{2} + \frac{v_{2}^{2}}{2g}\right) \qquad \dots (iv)$$

$$= \left(d_{1} + \frac{q^{2}}{2gd_{1}^{2}}\right) - \left(d_{2} + \frac{q^{2}}{2gd_{2}^{2}}\right)$$

$$= \frac{q^{2}}{2g} \left(\frac{1}{d_{1}^{2}} - \frac{1}{d_{2}^{2}}\right) - (d_{2} - d_{1}) = \frac{q^{2}}{2g} \cdot \frac{d_{2}^{2} - d_{1}^{2}}{d_{1}^{2}d_{2}^{2}} - (d_{2} - d_{1})$$

...(i)

But from equation (i),





The loss of energy head can be determined from equation (iv) or (v). The depths  $d_1$  and  $d_2$  on either side of the hydraulic jump are called sequent depths.

We know, that as a hydraulic jump is formed the area of flow suddenly increases. Just as in the case of pipes a loss of head occurs in a sudden enlargement, we find in the case of channel flow also a loss of head occurs due to sudden increase in the area of flow brought about by the hydraulic jump.

If no losses had taken place then the specific energy head would be the same for the depths  $d_1$  and  $d_2$  before and after the hydraulic jump. Due to loss of energy head the depth  $d_2$  reached is less than what would have been reached in the theoretical case.

At section 1-1 the specific energy head is  $E_1$ . If no loss of head occurs then the depth after the jump should have been  $E_1Q$ . See Fig. 14.85. But due to losses the specific energy head after the jump is  $E_2$  and the actual depth after the jump is  $d_2 = E_2Q'$ .

In the figure  $E_1E_2$  = loss of head  $h_1$  due to hydraulic jump.

...(v)



The expressions obtained in the theory of hydraulic jump presented above are based on the following assumptions:

- i) The bed of the channel is horizontal, i.e., bed slope i = 0
- (ii) Friction at bottom and sides of the channel is ignored.
- (iii) The velocity is uniform at the channel section.
- (iv) Depth wise pressure variation is hydrostatic.
- (v) The hydraulic jump occurs abruptly.

Hydraulic jumps may be classified based on Fraud's number Fr₁ upstream of the jump as given in the table below:

Range of Fr <sub>1</sub>	Description of the hydraulic jumps	Classification	
Equal to 1 1 - 1.7	No jump occurs. Flow is critical Surface undulations- No sharp or abrupt rise of water level seen	 Undular jump	
1.7-2.5	Jump is seen as a series of rollers on the surface.	Weak jump	
2.5 - 4.5	Surface shows oscillations without showing periodicity. Large surface waves on downstream side, Downstream bank channel likely to be damaged	Oscillatary jump	
<ul> <li>4.5 - 9 A clear distinct steady hydraulic jump is formed. There is considerable dissipation of energy due to the jump. The downsteam flow is smooth</li> </ul>		Steady jump	
Greater than 9	reater Powerful jump occurs. There is large dissipation of energy due to the jump. Downstream flow is rugged. Strong jum		



### Depth of Hydraulic jump as a Function of Froude Number:

We know that the depth of flow after the hydraulic jump is given by -

$$d_2 = \frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2v_1^2 d_1}{g}} = -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + \frac{8v_1^2}{gd_1}}$$

Froude number on the upstream side of jump

$$F_{r1} = F_{r1} = \frac{v_1}{\sqrt{gd_1}}$$
  

$$\therefore \qquad F_{r1}^2 = \frac{v_1^2}{gd_1}$$
  

$$\therefore \qquad d_2 = -\frac{d_1}{2} + \frac{d_1}{2}\sqrt{1 + 8F_{r1}^2}$$
  

$$\therefore \qquad d_2 = \frac{d_1}{2} \left(\sqrt{1 + 8F_{r1}^2} - 1\right)$$

(i) Froude number before jump is always greater than 1.

(ii) Froude number after the jump is always less than 1.

(iii) Higher the pre jump  $F_{r1}$  lower will be post jump  $F_{r2}$ .

(iv) The hydraulic jump is an irreversible and discontinuous process.

#### Height of the Standing Wave or Hydraulic Jump:

This is the difference of water levels between two sections before and after the hydraulic jump.

Height of standing wave =  $(d_2 - d_1)$ 

#### Length of Hydraulic Jump:

This cannot be calculated analytically. The exact point of commencement of the jump and the exact point where it ends are not well defined. For purposes of analysis we may assume the length of the hydraulic jump to be 5 to 7 times the height of the jump.

#### A hydraulic jump occurs in site in the following conditions:

(i) When water moving in shooting flow impacts with water having a larger depth with streaming flow.

(ii) On the downstream sides of sluices.

(iii) At the foot of spillways.

(iv) Where the gradient suddenly changes from a steep slope to a flat slope.

### **Relation between Pre Jump and Post Jump Froude Numbers:**

Let F<sub>r1</sub> and F<sub>r2</sub> be the Froude numbers before and after the hydraulic jump.

We know,  $F_{r1} = \frac{v_1}{\sqrt{gd_1}} \text{ and } F_{r2} = \frac{v_2}{\sqrt{gd_2}}$   $\therefore \qquad v_1 = F_{r1}\sqrt{gd_1} \text{ and } v_2 = F_{r2}\sqrt{gd_2}$ By continuity condition,  $d_1v_1 = d_2v_2$ 

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$$\frac{d_1 r_1}{d_1 r_1 \sqrt{g d_1}} = \frac{d_2 r_2}{d_2 F_{r_2} \sqrt{g d_2}}$$
$$\left(\frac{d_1}{d_2}\right)^{3/2} = \frac{F_{r_2}}{F_{r_1}}$$
$$\frac{d_1}{d_2} = \left(\frac{F_{r_2}}{F_{r_1}}\right)^{2/3}$$

We also know,

 $d_{2} = \frac{d_{1}}{2} \left( \sqrt{1 + 8F_{r1}^{2}} - 1 \right)$  $\frac{d_{1}}{d_{2}} = \frac{2}{\sqrt{1 + 8F_{r1}^{2}} - 1}$ 

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From the above relations for  $\frac{d_1}{d_2}$ 

$$\left(\frac{F_{r2}}{F_{r1}}\right)^{2/3} = \frac{2}{\sqrt{1+8F_{r1}^2}-1}$$
$$\frac{F_{r2}}{F_{r1}} = \left[\frac{2}{\sqrt{1+8F_{r1}^2}-1}\right]^{3/2}$$
$$F_{r2} = F_{r1}\left[\frac{2}{\sqrt{1+8F_{r1}^2}-1}\right]^{3/2}$$

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For a hydraulic jump to occur, the pre jump Froude number  $F_{r1}$  should be greater than 1. The post jump Froude number  $F_{r2}$  will be less than 1.

The table below shows the values of the post jump Froude number  $F_{r_2}$  for various values of pre jump Froude number  $F_{r_1}$ .

#### Hydraulic Jump in a Triangular Channel:

Consider a triangular channel whose vertex angle is 20. Let the discharge in the channel be Q. Consider sections 1-1 and 2-2 before and after the hydraulic jump.



Let  $d_1$  and  $d_2$  be the depths of flow at these sections. Let  $v_1$  and  $v_2$  be the velocities at these sections. Area of flow at section  $1 - 1 = A_1 = d_1^2 \tan \theta$ Area of flow at section  $2-2 = A_2 = d_2^2 \tan \theta$  $=wA_{1}\overline{y}_{1} = wd_{1}^{2} \tan \theta \frac{d_{1}}{3} = \frac{wd_{1}^{3}}{3} \tan \theta$  $=wA_{2}\overline{y}_{2} = wd_{2}^{2} \tan \theta \frac{d_{2}}{3} = \frac{wd_{2}^{3}}{3} \tan \theta$  $=P_{2} - P_{1} = \frac{w}{3} \tan \theta (d_{2}^{3} - d_{1}^{3})$ 

Pressure force at section 1-1

Pressure force at section 2-2

Net retarding force

But net force = mass flowing per second × change in velocity

$$= \frac{wQ}{g}(v_1 - v_2) = \frac{wQ}{g} \left( \frac{Q}{d_1^2 \tan \theta} - \frac{Q}{d_2^2 \tan \theta} \right)$$
$$= \frac{wQ^2}{g \tan \theta} \left( \frac{1}{d_1^2} - \frac{1}{d_2^2} \right) = \frac{wQ^2(d_2^2 - d_1^2)}{g \tan \theta d_1^2 d_2^2}$$
$$\therefore \qquad \frac{w}{3} \tan \theta (d_2^3 - d_1^3) = \frac{wQ^2(d_2^2 - d_1^2)}{g \tan \theta d_1^2 d_2^2}$$
$$\therefore \qquad d_2^3 - d_1^3 = \frac{3Q^2(d_2^2 - d_1^2)}{g d_1^2 d_2^2 \tan^2 \theta}$$

When the discharge Q and the depth d<sub>1</sub> before the hydraulic jump are known we can determine the depth  $d_2$  after the hydraulic jump by solving the above equation in  $d_2$  by trial and error.

## **Flow Profile**

The water surface profile is a measure of how the flow depth changes longitudinally. The profiles are classified based on the relationship between the actual water depth (y), the normal depth  $(y_n)$  and the critical depth  $(y_c)$ . Normal depth is the depth of flow that would occur if the flow was uniform and steady, and is usually predicted using the Manning's

<u>Equation.</u> Critical depth is defined as the depth of flow where energy is at a minimum for a particular discharge.

Flow profiles are classified by the slope of the channel (So),  $y_n$ , and  $y_c$ . There are five slope classifications designated by the letters C, M, S, A, and H (critical, mild, steep, adverse, and horizontal) respectively.

- Mild (M) if  $y_n > y_c$
- Steep (S) if y<sub>n</sub> < y<sub>c</sub>
- Critical (C) if y<sub>n</sub>= y<sub>c</sub>
- Adverse (A) if So < 0 (if slope is positive in the downstream direction)
- Horizontal (H) if So = 0

The profile is further classified according to the relative position of the actual flow depth to normal and critical depth as designated by the numbers 1, 2, and 3.

- Type 1 curve: Actual depth is greater than y<sub>c</sub> and y<sub>n</sub>, flow is subcritical
- Type 2 curve: actual depth is between  $y_c$  and  $y_n$ , flow can be either subcritical or supercritical
- Type 3 curve: actual depth is less than both y<sub>c</sub> and y<sub>n</sub>, flow is supercritical.

**Subcritical** occurs when the actual water depth is greater than critical depth. Subcritical flow is dominated by gravitational forces and behaves in a slow or stable way. It is defined as having a Froude number less than one.

**Supercritical** flow is dominated by inertial forces and behaves as rapid or unstable flow. Supercritical flow transitions to subcritical through a hydraulic jump which represents a high energy loss with erosive potential. When the actual depth is less than critical depth it is classified as supercritical. Supercritical flow has a Froude number greater than one.

**Critical** flow is the transition or control flow that possesses the minimum possible energy for that flowrate. Critical flow has a Froude number equal to one.

#### **Flow Profile Classification**

<b>Critical Slope</b> is one that sustains uniform, critical flow $(y_n=y_c)$ . Critical flow is unstable and a small fluctuation in energy will shift the flow into supercritical or subcritical flow.	



## Assignment 1

**Q1** Derive the geometric conditions for most economical section of a rectangular, trapezoidal, triangular and circular channel section.

**Q2** A trapezoidal channel is required to carry 6m<sup>3</sup>/s of water at velocity of 5.4km/hr. Find the most economical cross section if channel has a side slopes. 1 vertical to 2 horizontal. For the same discharge what saving in power would result if this trapezoidal section is replaced by a rectangular section 1.5m deep and 3m wide. C= 55 in Chezy's formula

**Q3** A 3m wide rectangular channel conveys  $12m^3/s$  of water at a depth of 2m. Calculate (i) specific energy and the conjugate depth (ii) critical depth, critical velocity and the minimum specific energy. Also compute the Froude number and comment on the nature of flow, i.e. whether subcritical or supercritical.

**Q4** A triangular gutter, whose sides include an angle of 60° conveys water at a uniform depth of 25cm. If the discharge is  $0.04m^3/s$  workout the bed gradient of the trough. Use the Chezy formula assuming that C = 52 m<sup>1/2</sup>/s

**Q5** A rectangle channel conveys a discharge of 2.85m<sup>3</sup>/s per m width at a depth of 1.5m .Find the minimum rise in the bed level at a section in order to produce critical flow. For this condition find also the fall in the level of the water surface.

**Q6** A 3m wide rectangular channel conveys  $12m^3/s$  of water at a depth of 2m. Calculate (i) specific energy and the conjugate depth (ii) critical depth, critical velocity and the minimum specific energy. Also compute the Froude number and comment on the nature of flow, i.e. whether subcritical or supercritical.

**Q7** The loss of energy head in a hydraulic jump is 4.25m. The Froude number just before the jump is 7.5. Find (i) discharge per meter width of channel (ii) the depth before and after the hydraulic jump (iii) Froude number after the jump. (iv) Percentage loss of energy head due to jump. (v) Length of the jump

**Q8** What is a venturiflume? State the equation giving discharge through such a flume.

**Q9** A discharge of 18m<sup>3</sup>/s flows through a rectangular channel 6m wide at a depth of 1.6m Find (i) The specific energy (ii) The critical depth (iii) Sate whether the flow is subcritical or super critical (iv) Conjugate depth

**Q10** Obtain for critical flow condition a relation between the critical depth and the specific energy for a trapezoidal channel of bottom width b and side slope 1 vertical to n horizontal